According to Fig. 11 a plot of the deviation Δ versus $\ln T$ for the samples *Au*-2.1Co and *Pt*-9.07Rh gives a straight line between about 30 and 200°K. As has been mentioned in Sec. IV, in our *Au*-Co alloy the effective Co concentration may be appreciably smaller than 2.1 at.%. This may explain why we did not find negative deviations Δ as has been reported by others for this system.[51] The linearity of both curves shown in Fig. 11 may suggest that between about 30 and 200°K the Kondo term in the resistivity proportional to $\ln T$ is the dominant contribution to Δ. The tailing off in the curves at lower temperatures may be associated with a small contribution to Δ from another mechanism. An interpretation of the curves in Fig. 11 with Kondo's $\ln T$ term would suggest that the *s-d* exchange integral J is positive in both samples. From the slope of the curve shown in Fig. 11, in combination with Eqs. (35) and (36) the value $J=0.6$ eV is then obtained for the sample *Au*-2.1 Co.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge valuable suggestions from R. E. Govednik regarding the experimental procedures. The authors had helpful discussions with C. van Baarle, J. E. Robinson, and O. C. Simpson.

PHYSICAL REVIEW B VOLUME 1, NUMBER 8 15 APRIL 1970

Transition Radiation from an Irradiated Multilayer

S. Y. Shieh

Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37916

(Received 29 December 1969)

A general formula for the transition radiation emitted by a uniformly moving charged particle normally incident on a multilayer, either self-supporting or deposited on a thick substrate, is calculated. This formula enables one to take into account the exact multilayer configuration of targets, e.g., oxide layers in the analysis of transition radiation data.

1. INTRODUCTION

Transition radiation, emitted by uniformly moving charged particles crossing the interface of two media having different dielectric properties, has recently been the subject of many theoretical and experimental studies.[1,2] More recently the construction of a new high-energy particle detector making use of transition radiation is being developed.[3] In experimental work, the targets often consist of multilayers, either self-supporting or deposited on a thick substrate, but a general formula which can account for the real configuration of multilayers has not yet been calculated. In this paper, we present the result of an exact treatment of the transition radiation from an arbitrary multilayer due to normally incident charged particles. Since an excellent exposition of the underlying electromagnetic theory and the methods used below is available in Ref. 4, we restrict ourselves to the essential points and the new results in the following concise presentation of the derivation of a general formula for transition radiation from a multilayer.

[1] F. G. Bass and V. M. Yakovenko, Usp. Fiz. Nauk. **86**, 189 (1965) [English transl.: Soviet Phys.—Usp. **8**, 420 (1965)]. This is the most comprehensive review article up to 1965.

[2] J. C. Ashley, L. S. Cram, and E. T. Arakawa, Phys. Rev. **160**, 313 (1967).

[3] L. C. L. Yuan, C. L. Wang, and S. Prünster, Phys. Rev. Letters **23**, 496 (1969).

[4] R. H. Ritchie and H. B. Eldridge, Phys. Rev. **126**, 1935 (1962).

2. GEOMETRY OF PROBLEM AND BOUNDARY CONDITIONS

Consider an n layer bounded by the $n+1$ plane interfaces parallel to the xy plane located at $z=D_p$ ($p=1, \ldots, n+1$). The thickness d_p of the pth layer between $z=D_p$ and $z=D_{p+1}$ is $d_p=D_{p+1}-D_p$, and its dielectric constant is ϵ_p. The semi-infinite spaces $z\leq D_1=0$ and $z\geq D_{n+1}$ are characterized by dielectric constants ϵ_0 and ϵ_{n+1}, respectively.

When a uniformly moving charged particle passes through the n layer along the normal to the plane interfaces of the n layer, taken as the z axis, the only nonvanishing component of the current density $\mathbf{j}(\mathbf{r},t) = (0,0j_z)$ and the Hertz vector $\mathbf{\Pi}(\mathbf{r},t)=(0,0,\Pi_z)$ will be the z component and we omit the subscript z below. If the particle carries a charge Ze and moves at velocity v, then the current density is given by

$$j(\mathbf{r},t)=Zev\delta(x)\delta(y)\delta(z-vt). \tag{1}$$

Here we have assumed that the particle crosses the front interface $z=D_1=0$ at time $t=0$ and $\delta(x)$ denotes Dirac's δ function.

The Fourier transform of (1),

$$J(k_x,k_y,\omega|z)=\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}dy\int_{-\infty}^{+\infty}dt \times j(\mathbf{r},t)e^{-i(xk_x+yk_y-\omega t)}, \tag{2}$$

can be found easily and one gets

$$J(k_x,k_y,\omega|z)=\frac{Ze}{(2\pi)^{3/2}}e^{i\eta z}, \quad (3)$$

where the abbreviation $\eta=\omega/v$ was used.

The Fourier transform of the Hertz vector in the pth layer is related to the current density by the equation

$$\left(\frac{d^2}{dz^2}+q_p^2\right)\Pi_p(k_x,k_y,\omega|z)=\frac{4\pi}{i\omega\epsilon_p}J(k_x,k_y,\omega|z), \quad (4)$$

in which $q_p^2=(\epsilon_p\omega^2/c^2)-\kappa^2$ with $\kappa^2=k_x^2+k_y^2$. The general solution of Π_p is given by

$$\Pi_p=A_pe^{iq_pz}+A_p'e^{-iq_pz}+\Gamma\frac{1}{\epsilon_p}\frac{e^{i\eta z}}{q_p^2-\eta^2}, \quad (5)$$

with $\Gamma=(Ze/(2\pi)^{3/2})4\pi/i\omega$. If q_p denotes the root $+[(\epsilon_p\omega^2/c^2)-\kappa^2]^{1/2}$ with $\mathrm{Im}q_p>0$, then A_p and A_p' represent the amplitudes of the free waves in the pth layer propagating in the positive and negative direction of the z axis, respectively. The Hertz vectors in the semi-infinite spaces $z\leq D_1=0$ and $z\geq D_{n+1}$ have the following forms:

$$\Pi_0=A_0'e^{-iq_0z}+\Gamma\frac{1}{\epsilon_0}\frac{e^{i\eta z}}{q_0^2-\eta^2} \quad (6)$$

and

$$\Pi_{n+1}=A_{n+1}e^{iq_{n+1}z}+\Gamma\frac{1}{\epsilon_{n+1}}\frac{e^{i\eta z}}{q_{n+1}^2-\eta^2}, \quad (7)$$

because there exist only those waves which move away from the n layer in these two semi-infinite regions.

The boundary conditions

$$\epsilon_p\Pi_p=\epsilon_{p+1}\Pi_{p+1} \quad \text{and} \quad d\Pi_p/dz=d\Pi_{p+1}/dz$$

at the first, the pth, and the $(n+1)$th interfaces are given by Eqs. (8), (9), and (10), respectively:

$$\epsilon_0A_0'+\frac{\Gamma}{q_0^2-\eta^2}=\epsilon_1A_1+\epsilon_1A_1'+\frac{\Gamma}{q_1^2-\eta^2}, \quad (8a)$$

$$-q_0A_0'+\frac{\eta}{\epsilon_0}\frac{\Gamma}{q_0^2-\eta^2}=q_1A_1-q_1A_1'+\frac{\eta}{\epsilon_1}\frac{\Gamma}{q_1^2-\eta^2}, \quad (8b)$$

$$\epsilon_{p-1}(A_{p-1}e^{iq_{p-1}D_p}+A_{p-1}'e^{-iq_{p-1}D_p})+\frac{\Gamma e^{i\eta D_p}}{q_p^2-\eta^2}$$
$$=\epsilon_p(A_pe^{iq_pD_p}+A_p'e^{-iq_pD_p})+\frac{\Gamma e^{i\eta D_p}}{q_p^2-\eta^2}, \quad (9a)$$

$$q_{p-1}(A_{p-1}e^{iq_{p-1}D_p}-A_{p-1}'e^{-iq_{p-1}D_p})+\frac{\eta}{\epsilon_{p-1}}\frac{\Gamma e^{i\eta D_p}}{q_{p-1}^2-\eta^2}$$
$$=q_p(A_pe^{iq_pD_p}-A_p'e^{iq_pD_p})+\frac{\eta}{\epsilon_p}\frac{\Gamma e^{i\eta D_p}}{q_p^2-\eta^2}, \quad (9b)$$

$$\epsilon_n(A_ne^{iq_nD_{n+1}}+A_n'e^{-iq_nD_{n+1}})+\frac{\Gamma e^{i\eta D_{n+1}}}{q_n^2-\eta^2}$$
$$=\epsilon_{n+1}A_{n+1}e^{iq_{n+1}D_{n+1}}+\frac{\Gamma e^{i\eta D_{n+1}}}{q_{n+1}^2-\eta^2}, \quad (10a)$$

$$q_n(A_ne^{iq_nD_{n+1}}-A_n'e^{-iq_nD_{n+1}})+\frac{\eta}{\epsilon_n}\frac{\Gamma e^{i\eta D_{n+1}}}{q_n^2-\eta^2}$$
$$=q_{n+1}A_{n+1}e^{iq_{n+1}D_{n+1}}+\frac{\eta}{\epsilon_{n+1}}\frac{\Gamma e^{i\eta D_{n+1}}}{q_{n+1}^2-\eta^2}. \quad (10b)$$

Solving the above equations for A_0' and A_{n+1}, we obtain

$$A_0'=\frac{B\bar{q}-B'\bar{\epsilon}}{\epsilon_0\bar{q}+\bar{\epsilon}q_0} \quad (11a)$$

and

$$A_{n+1}e^{iq_{n+1}D_{n+1}}=\frac{Bq_0+B'\epsilon_0}{\epsilon_0\bar{q}+\bar{\epsilon}q_0}. \quad (11b)$$

Here the following abbreviations are used:

$$\begin{pmatrix}B\\B'\end{pmatrix}=L_1L_2\cdots L_nM_{n+1}\begin{pmatrix}B_{n+1}\\B_{n+1}'\end{pmatrix}+L_1L_2\cdots L_{n-1}M_n\begin{pmatrix}B_n\\B_n'\end{pmatrix}$$
$$+\cdots+L_1M_2\begin{pmatrix}B_2\\B_2'\end{pmatrix}+M_1\begin{pmatrix}B_1\\B_1'\end{pmatrix},$$

$$M_{p+1}=\frac{1}{2\epsilon_pq_p}\begin{pmatrix}q_p & \epsilon_p\\ q_p & -\epsilon_p\end{pmatrix} \quad \text{for} \quad p=1,\cdots,n$$

$$\text{and} \quad M_1=\begin{pmatrix}1&0\\0&1\end{pmatrix},$$

$$L_p=\frac{1}{2\epsilon_{p-1}q_{p-1}}\begin{pmatrix}a_p/\alpha_p & b_p\alpha_p\\ b_p/\alpha_p & a_p\alpha_p\end{pmatrix} \quad \text{for} \quad p=2,\cdots,n$$

$$\text{and} \quad L_1=\begin{pmatrix}\epsilon_1/\alpha_1 & \epsilon_1\alpha_1\\ q_1/\alpha_1 & -q_1\alpha_1\end{pmatrix},$$

$$a_p=\epsilon_pq_{p-1}+\epsilon_{p-1}q_p, \quad b_p=\epsilon_pq_{p-1}-\epsilon_{p-1}q_p,$$
$$\alpha_p=\exp(iq_pd_p),$$

$$\begin{pmatrix}B_p\\B_p'\end{pmatrix}=\frac{(\omega/c)^2(\epsilon_{p-1}-\epsilon_p)e^{i\eta D_p}\Gamma}{\epsilon_{p-1}\epsilon_p(q_{p-1}^2-\eta^2)(q_p^2-\eta^2)}$$
$$\times\begin{pmatrix}\epsilon_p\epsilon_{p-1}\\ \eta[\epsilon_p+\epsilon_{p-1}-(\eta^2+k^2)/(\omega/c)^2]\end{pmatrix},$$

and

$$\begin{pmatrix}\bar{\epsilon}\\ \bar{q}\end{pmatrix}=-\prod_{p=1}^{n}L_pM_{n+1}\begin{pmatrix}\epsilon_{n+1}\\ q_{n+1}\end{pmatrix}.$$

3. POYNTING FLUX AT LARGE DISTANCES

Having obtained the Fourier transform of the Hertz vector we can perform the inverse integration

$$\Pi(\mathbf{r},t)=\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}dk_x\int_{-\infty}^{+\infty}dk_y\int_{-\infty}^{+\infty}d\omega \times\frac{Ze}{(2\pi)^{3/2}}\frac{4\pi}{i\omega}A_0'(\text{or } A_{n+1})e^{i(xk_x+yk_y+zq_0-\omega t)}, \quad (12)$$

and in the far radiation zone the application of the saddle-point method yields

$$\Pi(\mathbf{r},t)=-\frac{Ze\cos\theta}{\pi c}\int_{-\infty}^{+\infty}d\omega\, A_0'(\text{or } A_{n+1})\frac{e^{i(kr-\omega t)}}{r}. \quad (13)$$

We have taken out the factor

$$\Gamma=\frac{Ze}{(2\pi)^{3/2}}\frac{4\pi}{i\omega}$$

of A_0' or A_{n+1} in (12) and set ϵ_0 or $\epsilon_{n+1}=1$, so that $k=|\mathbf{k}|=\omega/c$. In (13), $\mathbf{r}=(r\sin\theta\cos\phi,\ r\sin\theta\sin\phi,\ r\cos\theta)$, and one puts $\kappa=(\omega/c)\sin\theta$ into the expressions A_0' and A_{n+1}.

Since $\mathbf{E}=(\omega^2/c^2)\mathbf{\Pi}$, one gets in polar coordinates

$$E_0=H_\varphi=-\frac{Ze\cos\theta}{\pi c}\int_{-\infty}^{+\infty}d\omega\left(\frac{\omega}{c}\right)^2 \times A_0'\sin\theta(\text{or } -A_{n+1}\sin\theta)\frac{e^{i(kr-\omega t)}}{r}$$

and

$$E_\varphi=-H_\theta=0.$$

Therefore, the emitted radiation is polarized in the plane of emission and its Poynting flux is given by

$$S=\frac{c}{4\pi}\int_{-\infty}^{+\infty}E_\theta H_\varphi\,dt\,r^2\,d\Omega$$

$$=\frac{(Ze\cos\theta)^2}{\pi^2 c}\int_0^\infty\left(\frac{\omega}{c}\right)^4|A_0'(\text{or } A_{n+1})|^2\sin^2\theta\,d\Omega.$$

Finally, we obtain for the number of photons emitted per unit frequency interval at frequency ω and per unit solid angle in the direction θ with respect to the normal of the multilayer:

$$\frac{d^2N}{d\omega d\Omega}=\frac{e^2}{\hbar c}\frac{Z^2\cos^2\theta\sin^2\theta}{\pi^2\omega}\left(\frac{\omega}{c}\right)^4|A_0'|^2(\text{or }|A_{n+1}|^2). \quad (14)$$

One should note that $(\omega/c)^4|A_0'|^2$ is dimensionless and by multiplying the Eqs (11) by $(\omega/c)^2$ one can make all quantities involved in these fundamental formulas dimensionless and thus simplify the programming of these formulas. Furthermore, one can set ϵ_{n+1} (or ϵ_0) equal to 1 and get the forward (or backward) yield but the other ϵ_0 (or ϵ_{n+1}) can be left arbitrary so that the above formula can be applied to a multilayer deposited on a thick substrate.